Loewner Evolution and Stochastic Differential Equations

Hülya Acar[a,b], *Alexey Lukashov*[a,c]
[a]*Department of Mathematics, Fatih University, Turkey*
[b]*hulyaacar98@gmail.com* , [c]*alukashov@fatih.edu.tr*

Abstract: The classical Loewner equation in the unit disc is the following differential equation;

$$\begin{cases} \frac{d\Phi_t(z)}{dt} = G(\Phi_t(z), t) & \text{for almost every } t \in [0, \infty) \\ \Phi_0(z) = z \end{cases} \tag{1}$$

where $G(w,t) = -wp(w,t)$ with the function $p : \mathbb{D} \times [0,\infty) \to \mathbb{C}$ measurable in t, holomorphic in z, $p(0,t) = 1$ for all $t \geq 0$ and $\operatorname{Re} p(z,t) \geq 0$.

Recently Georgy Ivanov and Alexender Vasil'ev [5] considered version of this differential equation with

$$G(w,t) = \frac{(\tau(t)-w)^2 p(w,t)}{\tau(t)} \text{ . For } \tau(t,\omega) = \exp(ikB_t(\omega)). \tag{2}$$

They found a substitution which transform the Loewner equation to an Itô diffusion.

In this study we prove that under rather general substitutions on $\tau(t, B_t)$, it is a possible to find a substitution which transform (1) to an Itô diffusion if and only if $\tau(t, B_t)$ is given by (2).

Keywords: Loewner equation, diffusion, Loewner evolution.

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