Nonlinear matrix modeling of macro system asymptotic behavior

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Abstract: A class of the dynamical systems is proposed as nonlinear matrix models. Let F be the map of the form [1]

$$F: R^n \to R^n, \quad F\,y = \Phi(y)\,Ay, \qquad (1)$$

where R^n is n- dimensional real vector-space, $\Phi(y)$ is a scalar function, A is a matrix of n order. Select $X \subseteq R^n$, $FX \subseteq X$ then map F is in general, non-invertible and generates the dynamical system $\{F^m, X, Z_+\}$. Here F^m, $m \in Z_+$ is a cyclic semi-group of maps, X is a phase space of the system, Z_+ is the set of nonnegative integers. The systems $\{F^m, X, Z_+\}$ are used as mathematical models for determining the macro system dynamics in the presence of limiting factors. In the models, n is a number of macro system components, $y \in X$ is a vector of component characteristics, $X = \{y \in R^n | y \geq 0, \|y\| \leq a\}$, $a < \infty$ ($y \geq 0$ means nonnegativity of its coordinates), A is a matrix of component interrelations, $\Phi(y)$ is a limiting function (limiting factor).

There are some problems with the nonlinear matrix modeling one of which concerns reducing the system size. Since the number n may be very large then constructing appropriate algorithms for determining the system asymptotic behavior is essential. We constructed computer algorithm for determining the asymptotic behavior of macro systems governed by the systems $\{F^m, X, Z_+\}$. It is based on the results of qualitative theory which we develop for this class of the dynamical systems. Stabilized p- periodic macro system structure, the number of parameters ($\leq p$) by which the macro system dynamics is described, and the limit set of macro system component characteristics for any nonzero initial vector y are obtained by the algorithm. Here the macro system structure (at the time m) is a vector $\|F^m y\|^{-1} F^m y$ for any nontrivial trajectory $\{F^m y\}$. The algorithm is very useful at $p > n$ as well as at $p > n^2$.

Keywords: dynamical system, nonlinear matrix models, computer simulation.

References:

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