Hardy type inequalities involving suprema

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Abstract: Let $I = (a; b)$, $-\infty \le a < b \le \infty$, $1 \le p; q \le \infty$, w is a nonnegative and continuous function on I, and u^q, v^p are nonnegative locally integrable functions on I, and the function $v^{-p'}$ is also locally integrable on I.
Introduce the operators

$$R_\infty^+ f(x) = \sup_{a<t<x} w(t)|P_x^+ f(t)|, \quad R_\infty^- f(x) = \sup_{x<t<b} w(t)|P_x^- f(t)|,$$

where $P_z^+ f(t) = \int_t^z f(s)ds$, $P_z^- f(t) = \int_z^t f(s)ds$.
Consider the inequality

$$\|R_\infty^\pm f\|_{q,u} \le C^\pm \|f\|_{p,v} \qquad (1)$$

Inequality of type (1) has been studied in [1, 2, 3, 4], where the operators P_b^+, P_a^- were considered instead of the operators P_z^+, P_z^-, respectively. The more general case has been considered in [4], when the integral operators with the kernel satisfying "Oinarov condition" were considered instead of the operators P_b^+, P_a^-. Impact of the operators P_z^+, P_z^- can be written as integral operators considered in [4] but their kernels do not satisfy the "Oinarov condition".
We denote by

$$\Delta_w^+ g(x) = \sup_{a<t<x} w(t)|g(x) - g(t)|, \qquad \Delta_w^- g(x) = \sup_{x<t<b} w(t)|g(x) - g(t)|$$

the estimates with a weight w, the values deviation of the function g from $g(x)$, respectively, on the intervals $(a; x)$, $(x; b)$. If g is a locally absolutely continuous function on I with a derivative $g' \in L_{p,v}$, then the inequality (1) is equivalent to the inequality

$$\|\Delta_w^\pm g\|_{q,u} \le C^\pm \|g'\|_{p,v} . \qquad (2)$$

Here the necessary and sufficient conditions are obtained for the inequality (1), (2) when $1 \le p \le \le q \le \infty$, supplementing results of [5].
Keywords: Hardy type inequalities, Oinarov condition, integral operators.

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