Coercive estimates for solutions of one singular equation with the third-order partial derivative

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Abstract. Sufficient conditions of boundedness for the highest derivative of partial differential equations of the third order are established and coercive estimate in the norm of the space $C_*(\overline{\Omega},R)$ is obtained.

Let $\overline{\Omega}=[0,\omega]\times(-\infty,+\infty)$. We consider the equation

$$u_{xtt}=a_0(x,t)u_{xt}+a_1(x,t)u_x+a_2(x,t)u_t+a_3(x,t)u+f(x,t), \tag{1}$$

where the functions $a_i(x,t)\,(i=\overline{0,3})$, $f(x,t)$ are continuous and, generally speaking, are unbounded on $\overline{\Omega}$.

We denote by $C_*(\overline{\Omega},R)$ the space of bounded functions, which continuous with $t\in R$ at $x\in[0,\omega]$ and uniformly with respect $t\in R$ continuous for $x\in[0,\omega]$. Let $\|V(x,\cdot)\|_1=\sup\limits_{t\in R}\|V(x,t)\|$, where $\|V(x,t)\|=\max\limits_{i=\overline{1,n}}|V_i(x,t)|$. We investigate the properties for solutions $u(x,t)$ of equation (1) satisfying the conditions

$$u(0,t)=\psi(t),\ u(x,t),u_x(x,t),u_t(x,t),u_{xt}(x,t)\in C_*(\overline{\Omega},R). \tag{2}$$

Assume that $P_{\alpha,\beta}(x,t)=\dfrac{\alpha(x,t)}{\sqrt{\beta(x,t)}}$, $\theta(x,t)=\dfrac{1}{d}\int\limits_t^{t+d}a_1(x,\tau)d\tau.$

Theorem 1. Let the functions $a_i(x,t)$ $(i=\overline{0,3})$ of equation (1) be continuous on $\overline{\Omega}$ and ψ,ψ^*,ψ^{**} be continuous and bounded on R and the following conditions are satisfied:

a) $a_1(x,t)\geq\gamma>0$, γ - const., $\dfrac{a_1(x,t)}{a_1(x,\bar t)}\leq c$ at $t,\bar t\in R:|t-\bar t|<d,\ c,d$ - const.;

b) for every $\varepsilon>0$ there exists a number $\delta>0$ such that for all t from R and $x',x''\in[0,\omega]:|x'-x''|<\delta$ the inequality $\left|\dfrac{a_1(x',t)-a_1(x'',t)}{a_1(x'',t)}\right|<\varepsilon$ holds;

c) $$P_{a_0,a_1}(x,t)\leq K, P_{a_2,a_1}(x,t), P_{a_3,a_1}(x,t), P_{f,a_1}(x,t), \\ f(x,t),\sqrt{\theta(x,t)}\psi(t),\sqrt{\theta(x,t)}\psi^*(t)\in C_*(\overline{\Omega},R).$$

Then, there exists a unique solution $u(x,t)$ of problem (1), (2), moreover $u_{xtt}\in C_*(\overline{\Omega},R)$ and the following estimate

$$\|u_{xtt}\|_1+\|a_0u_{xt}\|_1+\|a_1u_x\|_1+\|a_2u_t\|_1+\|a_3u\|_1\leq C.$$

holds. Here, C depends on norms of the functions $f,\psi,$ and constants γ,K,c,d,ε.

References:

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