Spectral properties of some non strongly regular boundary value problems for fourth order differential operators

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Abstract: In this paper, we consider some class of not strongly regular [1] boundary value problems for fourth order differential operator, which is defined by the differential expression

$$y^{IV}(x) = \lambda y(x) \tag{1}$$

and with the two-point boundary conditions on the interval (0,1)

$$\begin{aligned}
&y'''(0) + a_{13}y''(0) + a_{14}y''(1) + a_{15}y'(0) + a_{16}y'(1) + a_{17}y(0) + a_{18}y(1) = 0,\\
&y'''(1) + a_{23}y''(0) + a_{24}y''(1) + a_{25}y'(0) + a_{26}y'(1) + a_{27}y(0) + a_{28}y(1) = 0,\\
&y''(0) + a_{35}y'(0) + a_{36}y'(1) + a_{37}y(0) + a_{38}y(1) = 0,\\
&\sqrt{2}y'(0) + y'(1) + a_{47}y(0) + a_{48}y(1) = 0.
\end{aligned} \tag{2}$$

Theorem 1. Eigenvalues of the problem (1)-(2) form two series $\lambda_{k,1}, \lambda_{k,2} : |\lambda_{k,1} - \lambda_{k,2}| \to 0, k \to \infty$, and eigenfunctions which correspond to these eigenvalues have the form

$$y_{k,1}(x) = \cos(\sqrt[4]{\lambda_{k,1}}x) - \sin(\sqrt[4]{\lambda_{k,1}}x) + o(\frac{1}{k}),$$

$$y_{k,2}(x) = \cos(\sqrt[4]{\lambda_{k,2}}x) - \sin(\sqrt[4]{\lambda_{k,2}}x) + o(\frac{1}{k}),$$

where k is sufficiently large integer.

Theorem 2. System of eigenfunctions $\{y_{k,1}(x), y_{k,2}(x)\}$ of the boundary-value problem (1)-(2) does not form a basis in $L_2(0,1)$.

The proof is based on the application of the necessary conditions for the basis in the Hilbert space [2].

Keywords: asymptotic formulas, eigenvalue, eigenfunction, not strongly regular boundary conditions.

References:

[1] M.A. Naimark, Lineinye differencialnye operatory (Linear Differential Operators), Nauka, Moskva, 2, 1969.

[2] M.A. Sadybekov, A.M. Sarsenbi, On a necessary condition for a system of normalized elements to be a basis in a Hilbert space, Vestn. Tomsk. Gos. Univ. Mat. Mekh., 13(1), pp. 44–46, 2013.