On estimates of M-term approximations of classes in a symmetric space

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Abstract: Symmetric space of periodic functions of many variables and Nikol`ski–Besov class in this space are considered. Inequalities of different metrics for trigonometric polynomials are proved. The exact order of the best M −term approximation of Nikol`ski–Besov class in symmetric space is obtained.

Let $I^m = \{\bar{x}\,(x_1, \dots, x_m) \in R^m;\ 0 \le x_j \le 1;\ j = 1, \dots, m\}$, $X(\varphi)$ −separable, symmetric space of Lebesgue with fundamental function φ defined in I^m. $\|f\|_X$ is the norm of element $f \in X(\varphi)$[1]. For function $\varphi(t)$, $t \in [0,1]$, suppose that

$$\alpha_\varphi = \underline{\lim}_{t\to 0} \frac{\varphi(2t)}{\varphi(t)}, \quad \beta_\varphi = \overline{\lim}_{t\to 0} \frac{\varphi(2t)}{\varphi(t)}.$$

Let $1 < \alpha_\varphi \le \beta_\varphi < 2$, $1 \le \theta \le \infty$ and $r > 0$. In the symmetric space Nikol`ski–Besov class $B^r_{X,\theta}$ is considered. The quantity $e_M(f)_X$ is the best M −term approximation of a function $f \in X(\varphi)$ [2]. For a given class $F \subset X(\varphi)$ let $e_M(F)_X = \sup_{f\in F} e_M(f)_X$. So, the following statement is true.

Theorem 1. Let $X(\varphi)$ − symmetric space and
$0 < \frac{1}{q} < \frac{1}{2} \le \log_2 \alpha_\varphi \le \log_2 \beta_\varphi < 1$, $1 < \tau < \infty$, $1 \le \theta \le \infty$. If
$\frac{1}{2} \le \frac{1}{q} < \log_2 \alpha_\varphi \le \log_2 \beta_\varphi < 1$ and $\log_2 \beta_\varphi - \frac{1}{q} < \frac{r}{m}$, then

$$e_M\left(B^r_{X,\theta}\right)_{L_{q,\tau}} \asymp \frac{M^{-\left(\frac{r}{m}+\frac{1}{q}\right)}}{\varphi(M^{-1})}.$$

If $0 < \frac{1}{q} < \log_2 \alpha_\varphi \le \log_2 \beta_\varphi \le \frac{1}{2}$ and $\frac{r}{m} > \frac{1}{2}$, then $e_M\left(B^r_{X,\theta}\right)_{L_{q,\tau}} \asymp M^{-\frac{r}{m}}$.

References:

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