On the solvability of the problem of the distributed optimal control of oscillation processes described by the Fredholm integro-differential equations

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Abstract: In this paper the problem of tracking were studying, where it is required to minimize the functional

$J[u(t)] = \int_0^T \int_Q [V(t,x) - \xi(t,x)]^2\, dxdt + \beta \int_0^T \int_Q p^2\,[t,x,u(t,x)]dxdt,$
$\beta > 0,$

on the set of solutions of the following boundary value problem

$V_{tt} - AV = \lambda \int_0^T K(t,\tau)V(\tau,x)\,d\tau + f[t,x,u(t,x)],\ \ x \in Q \subset R^n,\ \ 0 < t \le T,$
$V(0,x) = \psi(x),\quad x \in Q,$
$\Gamma V(t,x) \equiv \sum_{i,j=1}^n a_{ij}(x)V_{x_j}(t,x)cos(\delta,x_i) + a(x)V(t,x) = 0,$
$x \in \gamma,\ \ 0 < t \le T.$

Here, γ is a piecewise smooth boundary of the region Q, δ is a normal, which is conducted at the point $x \in \gamma$, A is an elliptic operator; $f[t,x,u(t,x)] \in H(Q_T)$, $Q_T = Q \times (0,T)$, $\psi(x) \in H(Q)$ are given functions, $u(t,x) \in H(Q_T)$ is distributed control, $K(t,\tau)$, $a(x)$, $a_{ij}(x)$ are known functions; T is a fixed moment of time; λ is a parameter. It is established that the optimal control $u = u^0(t)$ is defined as the solution of a nonlinear integral equation with discontinuous kernel and satisfies the additional conditions in the form of inequality. Solution is obtained in the form of a triplet $(u^0(t),\ V^0(t,x),\ J[u^0(t)])$, where $u^0(t)$ is the optimal control, $V^0(t,x)$ is the optimal process, $J[u^0(t)]$ is the minimal value of the functional.

Keywords: functional, Fredholm integro – differential equation, the optimality condition, nonlinear integral equation, optimal control.

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