On the solvability of the problem of the optimal boundary control of thermal processes described by the Volterra integro-differential equations

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Abstract: In this paper, we investigate the problem of tracking, where it is required to minimize the functional

$$J[u(t)] = \int_0^T \int_Q [V(t,x) - \xi(t,x)]^2 \, dxdt + \beta \int_0^T u^2(t)dt, \quad \beta > 0,$$

on the set of solutions of the following boundary value problem

$$V_t - AV = \lambda \int_0^t K(t,\tau)V(\tau,x)\, d\tau + g[t,x], \quad x \in Q \subset R^n, \ \ 0 < t \leq T,$$

$$V(0,x) = \psi(x), \quad x \in Q,$$

$$\Gamma V(t,x) \equiv \sum_{i,j=1}^{n} a_{ij}(x) V_{x_j}(t,x) cos(\delta, x_i) + a(x)V(t,x) = b[t,x]p[t,u(t)],$$

$$x \in \gamma, \ \ 0 < t \leq T.$$

Here, γ is a piecewise smooth boundary of the region Q, δ is a normal, which is conducted at the point $x \in \gamma$, A is an elliptic operator;

$$g[t,x] \in H(Q_T), \ \ Q_t = Q \times (0,T), b[t,x] \in H\big(\gamma \times (0,T)\big),$$

$$p[t,u(t)] \in H(0,T), \ \ \psi(x) \in H(Q)$$

are given functions, $u(t) \in H(0,T)$ is boundary control, $K(t,\tau)$, $a(x)$, $a_{ij}(x)$ are known functions; T is a fixed moment of time; λ is a parameter.

Keywords: Volterra integro – differential equation, nonlinear integral equation, optimal control.

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