Differentiable solutions of the second order elliptic equation

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Abstract: Let $n \geq 3$ and Q be a bounded set in R^n with a smooth boundary and Ω be an arbitrary open subset of Q. We study the second order elliptic equation with discontinuous intermediate coefficients:

$$-\Delta u + \sum_{k=1}^{n} a_k(x)u = f \quad , \tag{1}$$

where $x = (x_1, x_2, ..., x_n) \in \Omega$.

It is known that if $p > n$ and $a_k, f \in L_p(\Omega)$, $k = 1,2,...,n$, then there is a generalized solution $u \in W_p^1(\Omega)$ of equation (1) such that it has continuous first order partial derivatives in Ω (see [1], Ch. 3). This result can not be improved. Since if at least one of a_k $(k = 1,2,...,n)$ and f belongs to $L_n(\Omega) \setminus L_p(\Omega)$, $p > n$, then the solution u of equation (1) does not belong to $C_{loc}^{(1)}(\Omega)$. So the following question arises:

Question: Are there other spaces such that the above precise result about differentiable solutions of equation (1) holds for any a_k $(k = 1,2,...,n)$ and f in this spaces?

The purpose of this work is to answer this question.

Suppose that the variable coefficients a_k $(k = 1,2,...,n)$ and the right-hand side f of equation (1) belong to some space M. We find the necessary and sufficient conditions on M for continuous differentiability of the solution to equation (1), when M is a space of type F or a symmetric space, or one of a Sobolev and Besov space.

Keywords: elliptic equation, intermediate coefficient, Lorentz space.

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