A criterion for the strong solvability of the Neumann-Tricomi problem for the Lavrent'ev-Bitsadze equation in L_p

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Abstract: In this study, we consider the Neumann-Tricomi problem of finding $u(x,y)$ for the Lavrent'ev-Bitsadze equation with the following boundary value conditions

$$\begin{cases} Lu \equiv -sgny u_{xx} - u_{yy} = f(x,y), (x,y) \in \Omega, \\ \left.\dfrac{\partial u}{\partial n}\right|_{\sigma} = 0, \\ u|_{AC} = 0. \end{cases} \qquad (1)$$

Here, $\Omega \subset R^2$ is a finite domain bounded by a curve σ for $y > 0$ and by the characteristics $AC: x + y = 0$ and $BC: x - y = 1$ for $y < 0$, and the symbol $\frac{\partial}{\partial n}$ is the directional derivative in the outward normal to the σ.
The existence and uniqueness of regular solution of Neumann-Tricomi problem (1) was proved by Bitsadze [1]. The completeness of eigenfunctions of the Neumann-Tricomi problem for a degenerate equation of mixed type in the elliptic part of the domain was investigated by Moiseev and Mogimi [2].
Theorem 1. The Neumann-Tricomi problem (1) is strongly solvable for any right-hand side $f \in L_p(\Omega)$ if and only if

$$\alpha \le \frac{\pi}{8}\frac{p}{p-1}, \beta \le \frac{3\pi}{4}\frac{p}{p-1}.$$

Corollary 2. The Neumann-Tricomi problem in the classical domain, when the elliptic part of the domain coincides with the semi-circle, is not strongly solvable in $L_2(\Omega)$.
Keywords: Neumann-Tricomi problem, Lavrent'ev-Bitsadze equation, strong solution.

References:

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