The deficiency indices of singular differential operators in vector-valued functions space

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Abstract: In this work, we consider the minimal differential operator L_0 in the space $H = L^2(0,\infty)\oplus L^2(0,\infty)$ which is generated by the following differential expression:

$$L_0(y) = y^{(4)} + Q(x)y. \qquad (1)$$

Here $y(x)=(y_1(x),y_2(x))$, $0<x<\infty$ and $Q(x)=||q_{ij}||^2_{i,j=1}$ – is a real symmetric matrix, whose eigenvalues $\mu_1(x)\to+\infty$, $\mu_2(x)\to-\infty$, for $x\to\infty$.

Introduce $\varphi(x)=\frac{1}{2}arctg\frac{q_{22}-q_{11}}{2q_{12}}$. The function $\varphi(x)$ is called as the speed of the rotation of eigenvectors of the matrix $Q(x)$.

Theorem 1. Assume that for sufficiently large x_0 and $x > x_0$ the following inequalities

1) $|\varphi'(x)| < const$,

2) $0 < A \le \left|\frac{\mu_i(x)}{\mu_j(x)}\right| \le B,\ i,j = 1, 2,$

3) $\int_{x_0}^{\infty}\left|\mu_i^{-\frac{1}{4}}(x)\right|dx < \infty,\ \int_{x_0}^{\infty}\left|\frac{\mu_i'^2(x)}{\mu_i^{\frac{9}{4}}(x)} + \frac{\mu_i''(x)}{\mu_i^{\frac{5}{4}}(x)}\right|dx < \infty,\ \int_{x_0}^{\infty}\left|\frac{\varphi''(x)}{\mu_i^{\frac{1}{4}}(x)}\right|dx < \infty,\ i = 1,2$

4) $|\mu_i'(x)| \le C|\mu_i(x)|^{\alpha},\ C = const,\ i = 1,2,\ 0 < \alpha < \frac{5}{4}$

Are satisfied. Then, system (1) has eight linearly independent solutions $y_j(x,\lambda)$ *for* $x\to\infty$, *such that*

$$y_1 = \varphi_1(x,\lambda)\exp\left\{\int_0^x(\lambda-\mu_1(t))^{1/4}dt\right\}(1+o(1)),\ y_2 = \varphi_1(x,\lambda)\exp\left\{-\int_0^x(\lambda-\mu_1(t))^{1/4}dt\right\}(1+o(1)),$$

$$y_3 = \varphi_1(x,\lambda)\exp\left\{i\int_0^x(\lambda-\mu_1(t))^{1/4}dt\right\}(1+o(1)),\ y_4 = \varphi_1(x,\lambda)\exp\left\{-i\int_0^x(\lambda-\mu_1(t))^{1/4}dt\right\}(1+o(1)),$$

$$y_5 = \varphi_2(x,\lambda)\exp\left\{\int_0^x(\lambda-\mu_2(t))^{1/4}dt\right\}(1+o(1)),\ y_6 = \varphi_2(x,\lambda)\exp\left\{-\int_0^x(\lambda-\mu_2(t))^{1/4}dt\right\}(1+o(1)),$$

$$y_7 = \varphi_2(x,\lambda)\exp\left\{i\int_0^x(\lambda-\mu_2(t))^{1/4}dt\right\}(1+o(1)),\ y_8 = \varphi_2(x,\lambda)\exp\left\{-i\int_0^x(\lambda-\mu_2(t))^{1/4}dt\right\}(1+o(1)),$$

where

$$\varphi_1(x,\lambda)=\frac{1}{\sqrt[8]{(\lambda-\mu_1(x))^3}}\begin{pmatrix}\cos\varphi(x)\\ -\sin\varphi(x)\end{pmatrix},\ \varphi_2(x,\lambda)=\frac{1}{\sqrt[8]{(\lambda-\mu_2(x))^3}}\begin{pmatrix}\sin\varphi(x)\\ \cos\varphi(x)\end{pmatrix}.$$

Theorem 2. *Assume that conditions of Theorem 1 are satisfied. Then, the deficiency indices of operator* L_0 *are equal to* (6,6).

Keywords: differential operator, distribution of eigenvalues, indices of deficiency, asymptotic of the spectrum

References:

[1] M.A. Naimark, Linear differential operators, Nauka, Moscow, 1969.