The estimates of widths of classes in the Lorentz space

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Abstract: In this talk, Lorentz space with anisotropic norm (see [1]), of periodic functions of many variables is considered. The estimate of the Kolmogorov's widths $d_M(F,X)$, Nikol'ski-Besov-Amanov classes in the space Lorentz is obtained.

Let $\bar{p}=(p_1,...,p_m)$, $\bar{\theta}=(\theta_1,...,\theta_m)$ and $\theta_j, p_j \in (1,+\infty)$, $j=1,...,m$, $L^*_{\bar{p},\bar{\theta}}(I^m)$ - Lorentz space with anisotropic norm. $a_{\bar{n}}(f)$. Fourier coefficients of function $f \in L_1(I^m)$ with respect to system $\left\{e^{i\langle \bar{n},\bar{x}\rangle}\right\}_{\bar{n}\in Z^m}$. Suppose that $\langle \bar{y},\bar{x}\rangle=\sum_{j=1}^m y_j x_j$, $s_j=1,2,...$, $\rho(\bar{s})=\{\bar{k}=(k_1,...,k_m)\in Z^m : 2^{s_j-1}\le |k_j|<2^{s_j}, j=1,...,m\}$, $\delta_{\bar{s}}(f,\bar{x})=\sum_{\bar{n}\in\rho(\bar{s})} a_{\bar{n}}(f)e^{i\langle \bar{n},\bar{x}\rangle}$. In the Lorentz space $\overset{\circ}{L}{}^*_{\bar{p},\bar{\theta}}(I^m)$, consider Nikol'ski - Besov - Amanov class:

$$ {}^{\circ}S^{\bar{r}}_{\bar{p},\bar{\theta},\bar{\tau}}B=\left\{ f\in \overset{\circ}{L}{}^*_{\bar{p},\bar{\theta}}(I^m) : \left\| \left\{ 2^{\langle \bar{s},\bar{r}\rangle}\left\|\delta_{\bar{s}}(f)\right\|^*_{\bar{p},\bar{\theta}} \right\}_{s\in Z^m_+} \right\|_{l_{\bar{\tau}}} \le 1 \right\}, $$

where $\bar{\tau}=(\tau_1,...,\tau_m)$, $1<p_j<\infty$, $1\le\theta_j,\tau_j<+\infty$, $j=1,...,m$.

Theorem. Let $1<\theta^{(1)}_j,\theta^{(2)}_j$, $1\le\tau_j\le\infty$, $j=1,...,m$ and

$$0<r_1+\frac{1}{q_1}-\frac{1}{p_1}=...=r_v+\frac{1}{q_v}-\frac{1}{p_v}<r_{v+1}+\frac{1}{q_{v+1}}-\frac{1}{p_{v+1}}\le...\le r_m+\frac{1}{q_m}-\frac{1}{p_m}.$$

1. If $1<p_j\le 2<q_j, r_j>\frac{1}{p_j}, j=1,...,m,$ then

$$d_M\left(S^{\bar{r}}_{\bar{p},\bar{\theta}^{(1)},\bar{\tau}}B, L^*_{\bar{q},\bar{\theta}^{(2)}}\right)\le C\left(\frac{\log^{v-1}M}{M}\right)^{\left(r_1+\frac{1}{2}-\frac{1}{p_1}\right)}(\log M)^{\sum_{j=2}^{v}\left(\frac{1}{2}-\frac{1}{\tau_j}\right)_+}.$$

2. If, $2\le p_j<q<\infty, j=1,...,m, r_1>\beta=\dfrac{\frac{1}{p_1}-\frac{1}{q}}{1-\frac{2}{q}}$, then

$$d_M\left(S^{\bar{r}}_{\bar{p},\bar{\theta}^{(1)},\bar{\tau}}B, L^*_{q,\bar{\theta}^{(2)}}\right)\le C\left(\frac{\log^{v-1}M}{M}\right)^{r_1}(\log M)^{\sum_{j=2}^{v}\left(\frac{1}{2}-\frac{1}{\tau_j}\right)_+}.$$

Keywords: Lorentz space, Besov class, the Kolmogorov width.

References:

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