Periodic problem for first-order equations with deviating argument

Amir Shaldanbaev[a], *Assylzat Kopzhassarova*[b], *Aigul Imanbaeva*[c]
[a,b,c]*M. Auezov South-Kazakhstan State University,Kazakhstan,*
[a]*shaldanbaev51@mail.ru*, [b]*asyl_k@mail.ru*, [c]*aigul_baratovna@bk.ru*

Abstract: In the present work, we study spectral characteristic marginal problem:

$$y'(x) = \lambda y(\alpha - x), y(0) = y(2\pi),$$

where $0 \ll \alpha \leq 2\pi$, λ is a spectral parameter.
We consider in the space $L^2(0,2\pi)$ periodic problem

$$y'(x) = \lambda y(\alpha - x), \qquad (1)$$

$$y(0) = y(2\pi), \qquad (2)$$

where α is real value satisfying condition $0 \ll \alpha < 2\pi$, and λ is a spectral parameter.
Similar problem was considered in earlier work [1]

$$y'(x) = \lambda y(1 - x),$$

$$\alpha y(0) + \beta y(1) = 0$$

where α, β are complex numbers, λ is spectral parameter. In our case, a magnitude α does not coincide with the end of the interval, i.e. α ≠ 2π and this significantly affects the results. It turns out that the problem (1) - (2) has a complete and orthogonal system of eigenvectors and also another series of eigenvectors, which form an incomplete system in $L^2(0,2\pi)$. When $\alpha = 2\pi$ second series disappear and the results coincide with the results of work [1].
It is known that [2], the self-adjoint and quite continuous operator has a complete and orthogonal system of eigenvectors and it does not have other eigenvectors. Not completely continuous, but self-adjoint operator can have a complete orthogonal system of eigenvectors and real eigenvalues, corresponding to them.

Keywords: eigenvalues, eigenfunctions, boundary conditions, adjoined function.

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