Parameterization method for the second-order nonlinear three-point boundary value problem

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Abstract: In this study, we consider the three-point boundary value problem for second-order nonlinear differential equations

$$\begin{cases} x''(t) = f\big(t, x(t)\big), \qquad 0 < t < 1, \\ x'(0) = a \\ \beta_1 x(1) + \beta_2 x'(1) = \delta_1 x(\eta) + \delta_2 x'(\eta) + b, \\ 0 < \eta < 1, \end{cases} \tag{1}$$

where $f \in C([0,1] \times \mathbb{R}, \mathbb{R})$, and $a, b, \beta_1, \beta_2, \delta_1, \delta_2$ are real numbers.
Second-order three-point problems were arising at mathematical modeling of various processes of physics, chemistry, biology, ecology, economy, etc. Problem (1) was object of studying of many authors. For finding conditions of existence of solution the method of fixed points, the upper and lower solutions and monotone iterative method are applied (see [1-2] and the references therein). Despite numerous studies the questions of existence and qualitative properties of solutions of the three-point problems remain open. This leads to the development of effective methods for studying nonlinear three-point boundary value problems and construction of algorithms for finding their solutions.
In the present work, the problem (1) is investigated by parameterization method [3].
The conditions of existence of solutions to problem (1) are established by the terms of initial data. Algorithms of finding solution to problem (1) are proposed.
Keywords: second-order differential equation, three-point boundary value problem, parameterization method, solvability.
References:
[1] M.K. Kwong, J.S.W. Wong, Solvability of second-order nonlinear three-point boundary value problems, Nonlinear analysis, vol. 73, pp. 2343-2352, 2010.
[2] F. Li, J. Sun, M. Jia, Monotone iterative method for the second-order three-point boundary value problem with upper and lower solutions in the reversed order, Applied Mathematics and Computation, vol. 217, pp. 4840-4847, 2011.
[3] D.S. Dzhumabaev, Conditions the unique solvability of a linear boundary value problem for a ordinary differential equations, Computational Mathematics and Mathematical Physics, vol. 29, no. 1, pp. 34-46, 1989.