The positivity of second order difference operator with periodic conditions in Holder spaces

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Abstract: In this study, the second order of approximation of difference operator A_h^x approximates the second order differential operator A^x defined by the formula

$$A^x = -u_{xx}(x) + \delta u(x), \delta > 0$$

with domain

$$D(A^x) = \left\{u(x): u(x), u'(x), u''(x) \in C(R^1), u(x) = u(x + 2\pi), x \in R^1, \int_0^{2\pi} u(x)\,dx = 0\right\}$$

is considered. The Green's function of the difference operator A_h^x is constructed. The estimates for the Green's function are obtained. The positivity of operator A_h^x in the Banach space $C(R_{1h})$ of periodic mesh functions defined on R_{1h} is established. Here, $R_{1h} = \{x_k = kh, k = 0, \pm 1, \pm 2 \ldots\}$. It is proved that for any $\alpha \in \left(0, \frac{1}{2}\right)$, the norms in spaces $E_\alpha = E_\alpha(C(R_{1h}), A_h^x)$ and $C^{2\alpha}(R_{1h})$ are equivalent. The positivity of the operator A_h^x in the Holder spaces of $C^{2\alpha}(R_{1h})$, $\alpha \in \left(0, \frac{1}{2}\right)$ is proved.

Keywords: positivity of difference operators, fractional space, nonlocal boundary conditions, Green's function.

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