Characteristical determinant of the spectral problem for ordinary differential operator with boundary load

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Abstract: We consider problem for the differential equation:

$$l(u) = u^{(n)}(x) + q_2(x)v^{(n-2)}(x) + \cdots + q_n(x)u(x) = \lambda u(x), \quad 0 < x < 1 \tag{1}$$

with the integral boundary value conditions:

$$U_j(y) \equiv \sum_{k=0}^{n\text{-}1}\left[\alpha_{jk}u^{(k)}(0) + \beta_{jk}u^{(k)}(1)\right] + \int_0^1 \overline{p_0(x)}u(x)dx = 0, \quad j = \overline{1,n}, \tag{2}$$

where $p_{0j}(x) \in L_2(0,1)$.

We assume that coefficients of the equation: $q_k(x) \in C^{n\text{-}k}[0,1], \quad k = \overline{2,n}$; and forms $U_j(u)$ are linear independent and refer to strongly regular boundary value conditions.

Let L_1 be an operator in $L_2(0,1)$, given by the expression (1) and «perturbed» boundary value conditions:

$$U_j(u) = 0, \quad j = \overline{1,n}, \quad j \neq m,$$
$$U_m(u) = \int_0^1 \overline{p_m(x)}u(x)dx, \quad p_m(x) \in L_2(0,1). \tag{3}$$

This direction is closely connected with the study of operators with potential containing a delta function, but also has its peculiarities. Nowadays this direction is under the stage of accumulation of primary information, for which it is necessary to obtain formulas of the explicit form to compute eigenvalues and study their asymptotics. An important step in this direction is to construct explicitly or identify structure of the characteristic determinant of the spectral problem.

Keywords: spectral problem, operator with an integral perturbation, systems of root functions, strongly regular boundary conditions.

References:

[1] A.A. Shkalikov, On the basis of eigenfunctions of ordinary differential operators with integral boundary value conditions, Vestnik MGU, no. 6, pp. 12–21, 1982.
[2] M.A. Naimark, Linear Differential Operators, Moscow, 1969.