On solvability of some nonlocal boundary value problems with the Hadamard boundary operator

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Abstract: In the present work, questions of solvability of nonlocal problems for the Laplace equation in a $\Omega=\{x\in R^n : |x|<1\}$ - ball are studied.

Problem. To find a harmonic function $u(x)\in C(\bar{\Omega})$ such that the function $D^\alpha[u](x)$ is continuous in $\bar{\Omega}$ and satisfies the conditions:

$$u(x)+(-1)^k u(x^*)=f(x),\ x\in\partial\Omega_+,$$

$$D^\alpha[u](x)-(-1)^k D^\alpha[u](x^*)=g(x),\ x\in\partial\Omega_+.$$

Here, $k=0,1$, $f(x)\in C^{\lambda+\alpha}(\bar{\Omega}), g(x)\in C^{\lambda}(\bar{\Omega}), 0<\lambda,\ \lambda+\alpha$ is noninteger, D^α - operators of a fractional order in the Hadamard sense, any point $x=(x_1,x_2,...,x_n)\in\Omega$ we associate the "opposite" point $x^*=(\alpha_1 x_1,\alpha_2 x_2,...,\alpha_n x_n)\in\Omega$, here $\alpha_1=-1$, and $\alpha_j, j=2,...,n$ take one of the values ± 1.

It is proved correctness of the considered problem. Smoothness of the solution of considered problem is also studied in the Holder classes depending on the order of boundary operators.

It should be noted that problem is proposed and first studied in [1,2] for the case of integer $\alpha=1$.

Keywords: the Laplace equation, nonlocal problem, the Hadamard operator, solvability, smoothness.

References:

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