Well-posedness of a third order of accuracy difference scheme for Bitsadze-Samarskii type multi-point NBVPs

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Abstract: A multi-point nonlocal boundary value problem

$$\begin{cases} -u_{tt} - \sum_{r=1}^{n}\left(a_r(x)u_{x_r}\right)_{x_r} + \delta u(t,x) = f(t,x), \\ 0 < t < 1, \;\; x = (x_1, \dots, x_n) \in \Omega, \\ u_t(0,x) = \varphi(x), \;\; u_t(1,x) = \beta_1 u_t(\lambda_1,x) + \beta_2 u_t(\lambda_2,x) + \psi(x), \;\; x \in \overline{\Omega}, \\ \beta_1 + \beta_2 \leq 1, \;\; 0 \leq \lambda_1 < \lambda_2 \leq 1, \\ u(t,x) = 0, \;\; 0 \leq t \leq 1, \;\; x \in S, \;\; S = \partial\overline{\Omega} \end{cases} \tag{1}$$

for the multidimensional elliptic equation with Bitsadze-Samarskii condition is considered. Here, Ω is the unit open cube in

$$R^n = \{x = (x_1, \dots, x_2) : 0 < x_k < 1, 1 < k < n\}$$

with boundary S, $\overline{\Omega} = \Omega \cup S$ and $a_r(x) \geq 0 > 0$, $(x \in \Omega)$, $\varphi(x)$, $\psi(x)$ $(x \in \overline{\Omega})$, $f(t,x)$ $(t \in (0,1),\ x \in \Omega)$ are given smooth functions. δ is a large positive constant.

The third order of accuracy stable difference scheme for the approximate solution of (1) is presented. The stability, coercive stability and almost coercive stability estimates for the solution of this difference scheme are obtained. A numerical analysis is given to support the theoretical statements.

Keywords: Bitsadze-Samarskii problem, elliptic equation, nonlocal boundary value problems, difference scheme, well-posedness, stability, coercive stability.

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