About an approach to the fourth-order operator bundles

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Abstract: We consider the bundle form [1]

$$Au \equiv (\alpha^2 - 1)u^{(4)}(x) - 2\alpha\lambda^2 u''(x) + \lambda^4 u(x), \quad -1 < x < 1 \tag{1}$$

in the space $L_2(-1,1)$, where α is a complex number different from zero and ± 1, λ is spectral parameter. The expression (1) should be considered together with the spectral problem [2]

$$-u''(-x) + \alpha u''(x) = \lambda^2 u(x), \quad -1 < x < 1 \tag{2}$$

Here, $\alpha \neq 0$, $\alpha \neq \pm 1$ is a complex constant.

The first term in equation (2) contains involution. We assume that the spectral problem (2) is considered with some boundary conditions, the form of which we have not listed yet. There is following relation between the eigenfunctions of the spectral problem (2) and the operator bundle (1).

*Theorem 1***.** The eigenfunctions of the spectral problem (2) are the eigenfunctions of the operator bundle (1).

We consider the fourth-order operator bundle (1) with boundary conditions

$$u(-1) = 0, \qquad u(1) = 0, \quad u''(-1) = 0, \quad u''(1) = 0. \tag{3}$$

Theorem 2. The system eigenfunctions of the bundle (1), (3) contains subsystem

$$u_{k1}(x) = cos\left(\frac{\pi}{2} + k\pi\right)x, \;\; k = 0,1,2,\dots, u_{k2} = sink\pi x, \qquad k = 1,2,\dots,$$

which forms Riesz basis in the space $L_2(-1,1)$.

Keywords: operator bundle, eigenfunctions, basis, differential operator with involution.

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