Solvability of the nonlocal multi-point boundary value problem for system of quasilinear hyperbolic equations

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Abstract: In this study, we consider the multi-point boundary value problem on $\overline{\Omega} = [0,T] \times [0,\omega]$ for the second-order system of quasilinear hyperbolic equations

$$\begin{cases} \frac{\partial^2 u}{\partial x \partial t} = A(t,x)\frac{\partial u}{\partial x} + f\left(t,x,u,\frac{\partial u}{\partial t}\right), \\ u = (u_1,\dots,u_n) \in R^n, \ (t,x) \in \overline{\Omega}, \\ \sum_{i=0}^{m} P_i(x)\, u(t_i,x) = \varphi(x), \qquad x \in [0,\omega], \\ u(t,0) = \psi(t), \quad t \in [0,T], \end{cases} \tag{1}$$

where $n \times n$ matrix $A(t,x)$ is continuous on $\overline{\Omega}$, $n \times n$ matrices $P_i(x)$, n vector-function $\varphi(x)$ are continuously differentiable on $[0,\omega]$, $i = \overline{0,m}$, $0 = t_0 < t_1 < t_2 < \cdots < t_m = T$, n vector-function $\psi(t)$ is continuously differentiable on $[0,T]$. The initial data meet a coordination condition: $\sum_{i=0}^{m} P_i(0)\,\psi(t_i) = \varphi(0)$.

The necessary and sufficient conditions of a well-posed solvability of the linear problem corresponding to (1) in terms of the initial data were found in [1]. The main results of [1] based on equivalence of the well-posed solvability of the nonlocal boundary value problem for a system of hyperbolic equations and a family of multi-point boundary value problem for systems of ordinary differential equations. In the present work, the sufficient condition coefficients of the unique solvability of problem (1) are established by introducing additional functions [2] and applying results of concerning families of multi-point boundary value problems for system of ordinary differential equations, we suggest a solution algorithm.

Keywords: multi-point boundary value problem, hyperbolic equation, solvability.

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