Approximation problems in the variable exponent Lebesgue spaces

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Abstract: The variable exponent Lebesgue spaces are generalization of the classical Lebesgue spaces replacing the constant exponent p with a variable exponent function $p(\cdot)$. Interest in the variable exponent Lebesgue spaces has increased since 1990s, because of their use in the different application problems in mechanic, especially in fluid dynamic for the modeling of electrorheological fluids and also in the study of image processing and some physical problems (see, for example [3]). Nowadays there are sufficiently wide investigations relating to the fundamental problems of these spaces, in view of potential theory, maximal and singular integral operator theory and others. But the approximation problems in the variable exponent Lebesgue spaces, specially in the complex plane aren't investigated sufficiently wide.

In this study, we consider the direct and inverse problems of approximation theory in the variable exponent Lebesgue spaces of 2π periodic functions and also in the Smirnov classes of analytic functions defined on the simple connected domains of the complex plane. First, we obtain the direct and inverse theorems in the variable exponent Lebesgue spaces and later using the method developed in the papers [1] and [2] we investigate the similar problems in the variable exponent Smirnov classes and obtain the appropriate results in these spaces. For a given function f the approximation polynomials are constructed by using its series representation with respect to the Faber polynomials of the simple connected domain G, where f is defined. For the estimation of the approximation error are used the modulus of smoothness constructed via Steklov means of f. Using this modulus are also defined the generalized Lipschitz classes of analytic functions and in these classes the constructive characterization problems are investigated.

Keywords: direct theorem, inverse theorem, modulus of smoothness, Faber series, variable exponent Lebesgue spaces, Smirnov classes.

References:

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