Ill-posed problem for the biharmonic equation

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Abstract: Recently, among the experts on the equations of mathematical physics, there is considerably increased interest to problems that are ill-posed by J. Hadamard [1]. Problems of this kind are always attracted the attention of researchers. First of all, this is due not only to their importance in theory, but also to the fact that they have to be faced in many applications in various fields of science and technology. Due to the ill-posed problems it can be noted classical works of J. Hadamard [1], A.N. Tikhonov [2], M.M. Lavrent'ev [3] and many others, who paid the attention of researchers to the ill-posed problems and brought a significant contribution to the development of this important area of mathematics.

Formulation of the boundary value problem for the biharmonic equation is as follows. We consider the boundary value problem

$$\Delta^2 u = f\,,\ \{x,t\} \in Q\,; \tag{1}$$

$$u(0,y) = u_x(0,y) = 0\,,\ u(2\pi,y) = u_x(2\pi,y) = 0\,; \tag{2}$$

$$u(x,0) = 0\,,\ u_y(x,0) = \varphi_1(x)\,,\ u_{yy}(x,0) = 0\,,\ u_{yy}(x,1) = 0\,, \tag{3}$$

in the domain $Q = \{x, y \mid 0 < x < 2\pi,\ 0 < y < 1\}$ with the additional condition

$$u(x,1) = \psi(x) \in U_g \text{ is closed convex set of } H_0^{3/2}(0,2\pi)\,. \tag{4}$$

It is assumed that the data in problem (1)–(3) satisfies the following conditions

$$f \in (\tilde{H}^2(Q))'\,,\ \varphi_1 \in H_0^{1/2}(0,2\pi)\,,\ \psi \in H_0^{3/2}(0,2\pi)\,, \tag{5}$$

here

$$\tilde{H}^2(Q) = \left\{ u \mid u \in L_2(0,1;\ H_0^2(0,2\pi)),\ \frac{\partial^2 u}{\partial y^2} \in L_2(Q) \right\}.$$

In this paper, we use optimal control methods to solve problem (1)–(5).

Keywords: biharmonic equation, ill-posed problem, inverse problem, optimal control.

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