Fractional spaces generated by the positive differential operator in the half-line $\mathbb{R}^+$ and their applications

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Abstract: In this study, we investigate the structure of fractional spaces generated by the $2m$-th order differential operator

$$A^x = (-1)^m a(x) \frac{d^{2m}}{dx^{2m}} + \delta I$$

acting in the space of all continuous functions defined on the half-line $\mathbb{R}^+ = [0, \infty)$. Here, $\delta > 0$ is the sufficiently large number and $a(x)$ is a continuous function defined on $\mathbb{R}^+$ with $0 < s_1 \leq a(x) \leq s_2 < \infty$.
The domain of this operator is

$$D(A^x) = \left\{ u \in C^{2m}(\mathbb{R}^+) : u(0) = u'(0) = \cdots = u^{(m-1)}(0) = 0 \right\}.$$

Keywords: fractional spaces, positive operators, Green function.

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