On the stability of a time-dependent source identification problem

Allaberen Ashyralyev[a], *Ali Ugur Sazaklioglu*[b]
[a,b]*Department of Mathematics, Fatih University, 34500, Istanbul, Turkey*
[a]aashyr@fatih.edu.tr, [b]ugursazak@gmail.com

Abstract: In this study, the following time-dependent source identification problem subject to an integral overdetermined condition

$$\begin{cases} \frac{\partial u(t,x)}{\partial t} - \frac{\partial}{\partial x}\left(a(x)\frac{\partial u(t,x)}{\partial x}\right) + \delta u(t,x) = f(t,x) \\ +p(t)q(x), x \in (0,l), t \in (0,T), \\ u(0,x) = \varphi(x), x \in [0,l], \\ u_x(t,0) = u(t,l) = 0, t \in [0,T], \\ \int_0^l u(t,x)dx = \psi(t), t \in [0,T] \end{cases}$$

is considered. Here, (u,p) is the solution pair of the problem, $a(x)>0$, $f(t,x)$, $q(x)$, $\psi(t)$ and $\varphi(x)$ are given sufficiently smooth functions assuming $q'(0)=q(l)=0$, and $\int_0^l q(x)dx \neq 0$.

The stability inequalities for the solution of this problem are presented. Rothe and Crank-Nicholson difference schemes are constructed for the numerical solution of this problem. Moreover, stability and almost coercive stability inequalities for these difference schemes are given.

Keywords: finite difference method, integral condition, stability, almost coercivity.

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