Nonselfadjoint correct restrictions and extensions with real spectrum

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Abstract: In this work, we obtain sufficient conditions for the reality of the spectrum of some correct restrictions and extensions of operators. It is shown, that the system of eigenvectors forms a Riesz basis in the case of discrete spectrum.

We consider the densely defined minimal operator L_0 in Hilbert space. Let L_0 be a symmetric operator. Then $\hat{L} = L_0^*$ is the maximal operator (see [2]).

Using a known fixed boundary correct restriction L of maximal operator to describe all possible correct restrictions L_K of a maximal operator $\hat{L}$ in terms of an inverse operator was described in [2] as following

$$L_K^{-1} f = L^{-1} f + Kf, \quad \text{for every } f \in H, \qquad (1)$$

where K is an arbitrary bounded operator in Hilbert space H, satisfying the condition $R(K) \subset \ker \hat{L}$.

Theorem 1. Let L_0 be a symmetric linear operator, L- be a fixed selfadjoint correct extension of the operator L_0 in Hilbert space H. Then, if KL is a bounded operator and $I + KL > 0$ or $I + KL < 0$, then the correct restrictions L_K have only real spectrum where K is from (1).

Keywords: correct restriction, correct extension, Riesz basis.

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