

On a boundary-value problem with sewing condition of integral form for parabolic-hyperbolic equation with fractional derivative

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Abstract: In this study, we consider a boundary problem with sewing condition of integral form for mixed parabolic-hyperbolic type equation with the Caputo fractional derivative

$$0 = \begin{cases} u_{xx} - {}_c D_{0t}^\alpha u, & t > 0, \\ u_{xx} - u_t, & t < 0 \end{cases} \quad (1)$$

in the domain $\Omega = \Omega^+ \cup \Omega^- \cup AB$, where $0 < \alpha \leq 1$, $AB = \{(x, t) : 0 < x < 1, t = 0\}$,
 $\Omega^+ = \{(x, t) : 0 < x < 1, 0 < t < 1\}$, $\Omega^- = \{(x, t) : -t < x < t + 1, -1/2 < t < 0\}$,

${}_c D_{0t}^\alpha f = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-z)^{-\alpha} f'(z) dz$ is the Caputo fractional differential operator of the order α ($0 < \alpha \leq 1$), $\Gamma(\cdot)$ is Euler's gamma-function.

We prove the uniqueness of the solution for considered problem by the method of energy integrals. The existence of the solution have been proved by reducing the considered problem to the Fredholm integral equation. We represent solution in an explicit form using Green's function.

Boundary problems with sewing conditions of integral form were studied in works [1-3].

Keywords: fractional derivative, mixed type equation, sewing condition, Green's function.

References:

- [1] A.S. Berdyshev, A. Cabada, E.T. Karimov, N. Akhtaeva, On the Volterra property of a boundary problem with integral gluing condition for a parabolic-hyperbolic equation, *Boundary Value Problems*, 2013:94, 2013.
- [2] A.S. Berdyshev, A. Cabada, E.T. Karimov, On a nonlocal boundary problem for a parabolic-hyperbolic equation involving a Riemann-Liouville fractional differential operator, *Nonlinear Analysis*, vol. 75, pp. 3268-3273, 2012.
- [3] A.S. Berdyshev, E.T. Karimov, N. Akhtaeva, Boundary value problems with integral gluing conditions for fractional-order mixed-type equation, *International Journal of differential Equations*, Article ID 268465, 2011.