On the spectrum of Volterra integral equation with the "incompressible" kernel

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Abstract: We consider the singular Volterra integral equation of the second kind which has the "incompressible" kernel

$$\varphi(t)-\lambda\int_0^t K(t,\tau)\,\varphi(\tau)\,d\tau=f(t),\quad t>0, \tag{1}$$

where

$$K(t,\tau)=\frac{1}{2a\sqrt{\pi}}\left\{\frac{t+\tau}{(t-\tau)^{3/2}}\exp\left(-\frac{(t+\tau)^2}{4a^2(t-\tau)}\right)+\frac{1}{(t-\tau)^{1/2}}\exp\left(-\frac{t-\tau}{4a^2}\right)\right\}.$$

The feature of equation (1) follows from the limit relations for the kernel $K(t,\tau)$

$$\lim_{t\to 0}\int_0^t K(t,\tau)\,d\tau=1,\ \lim_{t\to+\infty}\int_0^t K(t,\tau)\,d\tau=1.$$

We assume that $|\lambda|>1$. Case $|\lambda|=1$ was considered in [1] and [2]. After allocating the characteristic part, we present equation as an equation with a difference kernel [3]. Further the initial equation is reduced to the Abel equation. As a result, it is proved the solvability of equation (1) for any function $f(t)$: $\sqrt{t}\,f(t)\in L(0;\infty)\cap C(0;\infty)$ and the presence of eigenfunctions which are found in explicit form. The equation has a continuous spectrum, and the multiplicity of the characteristic numbers grows with increasing $|\lambda|$.

Keywords: incompressible kernel, eigenfunction, Abel equation.

References:

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