On stability of a solution of the loaded heat equation

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Abstract: In this study, we consider the stabilization problem of finding $\{y(x,t),\ u_1(t),\ u_2(t)\}$ for the loaded heat equation

$$\begin{cases} y_t(x,t) - y_{xx}(x,t) + \alpha \cdot y(0,t) = 0, \quad \{x,t\} \in Q, \\ y\left(-\frac{\pi}{2}, t\right) = u_1(t), \ \ y\left(\frac{\pi}{2}, t\right) = u_2(t), \ \ y(x,0) = y_0(x), \end{cases} \tag{1}$$

where solution $y(x,t)$ tends to zero when $t \to \infty$, such that

$$\|y(x,t)\|_{L_2\left(-\frac{\pi}{2},\frac{\pi}{2}\right)} \leq C_0\, e^{-\sigma_0 t}. \tag{2}$$

Here $Q = \left\{x,t \mid -\frac{\pi}{2} < x < \frac{\pi}{2},\ \ t > 0\right\}$, $\alpha \in C$, σ_0 is a known positive number, $y_0(x) \in L_2(-\pi/2,\ \pi/2)$ is a given function. The equation (1) is called loaded ([1] – [3]).

For the boundary value problem (1) on the semibar of width π with the nonhomogeneous Dirichlet boundary conditions and initial condition on the segment $(-\pi/2,\ \pi/2)$ by the given function $y_0(x)$, we consider the auxiliary boundary value problem on the extended semibar of width 2π with the periodicity conditions (instead of Dirichlet conditions) and initial function $z_0(x)$ on the segment $(-\pi,\ \pi)$. Further, we determine a function $z_0(x)$ as continuation of the given function $y_0(x)$.

Theorem on solvability of the inverse problem (1) – (2) is proved and the algorithm of approximately construction of boundary controls is developed. Numerical calculations show efficiency of the offered algorithm.

Keywords: inverse problem, loaded heat equation, stabilization problem.

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