A new three-step iteration for generalized nonexpansive mappings in a CAT(0) space

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Abstract: Suzuki [1] introduced a condition on mappings.
Let T be a self mapping on a subset K of a metric space (X, d). Then, T is said to satisfy Condition (C) (sometimes T is called generalized nonexpansive mapping) if

$$\frac{1}{2}d(x,Tx) \leq d(x,y) \text{ implies } d(Tx,Ty) \leq d(x,y), \qquad \text{(C)}$$

for all $x, y \in K$.
Karakaya et. al. [2] established a new three-step iteration method in a Banach space as follows.

$$\begin{aligned} &x_1 \in K, \\ &z_n = (1-c_n)x_n + c_n T x_n, \\ &y_n = (1-a_n-b_n)z_n + a_n T z_n + b_n T x_n, \\ &x_{n+1} = (1-\alpha_n-\beta_n)y_n + \alpha_n T y_n + \beta_n T z_n, \end{aligned} \qquad (1)$$

where $\{a_n + b_n\}, \{\alpha_n + \beta_n\}, \{c_n\} \subset [0,1]$.
In this study, we apply the new three-step iteration process into a CAT(0) space and prove some theorems on the strong and Δ-convergence of the new three-step iteration for generalized nonexpansive mappings in a CAT(0) space.

Keywords: iteration methods, fixed point, CAT(0) space, nonexpansive mapping.

References

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