On boundary value problem with nonlocal conditions for the system of partial differential equations

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Abstract: In $\overline{\Omega} = \{(x,t): t \le x \le t+\omega, 0 \le t \le T\}$, $T > 0$, $\omega > 0$, we consider nonlocal boundary value problem for the system of partial differential equations

$$D\left[\frac{\partial}{\partial x}u(x,t)\right] = A(x,t)\frac{\partial u}{\partial x}(x,t) + S(x,t)u(x,t) + f(x,t)\,,\ (x,t) \in \overline{\Omega}, \quad (1)$$

$$B(x)\frac{\partial u}{\partial x}(x,0) + C(x)\frac{\partial u}{\partial x}(x+T,T) = d(x),\ x \in [0,\omega], \quad (2)$$

$$u(t,t) = \Psi_1(t),\quad t \in [0,T], \quad (3)$$

$$Du(t,t) = \Psi_2(t), t \in [0,T]. \quad (4)$$

Here, $D = \frac{\partial}{\partial t} + \frac{\partial}{\partial x}$; $u(x,t)$ is unknown n – vector function on $\overline{\Omega}$; $A(x,t)$, $S(x,t)$ are given $(n \times n)$ matrices functions, $f(x,t)$ is a known n – vector function on $\overline{\Omega}$; $B(x)$, $C(x)$ are given $(n \times n)$ matrices functions, $d(x)$ is a known n - vector – function on $[0,\omega]$; $\Psi_1(t)$, $\Psi_2(t)$ are given continuous functions on $[0,T]$.

A method of parameterization for the system of hyperbolic equations with mixed derivative was investigated in [1], [2].

In this work, new functions $v(x,t) = \frac{\partial u}{\partial x}(x,t)$, $w(x,t) = Du$ are introduced. Hence, the problem is reduced to the equivalent problem for the system of hyperbolic first – order equations with identical main part on Courant.

Sufficient conditions for the correct solvability of the problem in the terms of invertibility of the matrix, and boundary condition are obtained. In this work, an algorithm to find the solution of the problem is offered. Existence of the solution of this problem is established in the sense of Fridrihsu.

Keywords: nonlocal, correct solvability, hyperbolic, Courant, Fridrihsu.

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