Definition of number of the valid roots of the polynom

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Abstract: At calculation of members of a number of Sturm on the basis of computers loss of the importance of number and exponent disappearance are possible. It leads to loss of signs in a number of Sturm that attracts to the wrong definition of number of the valid roots of a polynom.

Problem: to find number of the valid roots of a polynom with the real coefficients.

$$f(p) = a_0 p^n + a_1 p^{n-1} + ... + a_{n-1} p + a_n \tag{1}$$

Use the following ranges of change of a variable in system (1) $-\infty \le p \le \infty$, $-\infty \le p \le 0$, $0 \le p \le \infty$ and $V(p)$- number of changes of signs in Sturm system interest. More effective and less labor-consuming method of creation of system of Sturm is offered. We will put $f_1(p) = f'(p)$. Then divide $f(p)$ on $f'(p)$ and the rest from this division, taken with the return sign, we accept for $f_2(p)$:

$$f_{k-2}(p) = f_{k-1}(p) g_{k-1}(p) - f_k(p), \tag{2}$$

then from the rest we take a derivative

$$f_{k+1}(p) = -f_k{}'(p), \tag{3}$$

where $f_0(p) = f(p)$, $f_1(p) = f'(p)$.

Steps (2), (3) we repeat for finding of polynoms f_{k-1}, f_k, $k = 2,3,...$ and so on until we will receive. In an existing method [1] operation (2) repeats until then yet we won't receive a constant $f_m(p) = const$. In an existing method [1] operation (2) repeats until then yet we won't receive a constant. The system of polynoms constructed by us meets all requirements of the theorem of Sturm

$$f(p) = f_0(p), f'(p) = f_1(p), f_2(p), ..., f_m(p) \tag{4}$$

Lemma. Differences $V(-\infty)$ - $V(\infty)$, $V(-\infty)$ - $V(0)$, $V(0)$ - $V(\infty)$ are equal to number of the valid, negative and positive roots of a polynom (1) respectively. Less labor-consuming new system (4) by definition of number of the valid roots of a polynom has smaller number of changes of signs, than in an existing method of Sturm.

Keywords: polynom, number of Sturm, roots, theorem of Sturm.

References:

[1] A.G. Kurosh, Kurs of the higher algebra, Moscow, Nauka, 1975.