# Spectrum structure of first order solvable pantograph differential operators

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**Abstract:** In this work, all solvable extensions of the minimal operator generated by linear pantograph-type delay differential-operator expression for first order in Hilbert space of vector-functions at finite interval in terms of boundary values have been described based on M.I. Vishik's result. Later on, the spectrum structure of these extensions has been researched.
**Keywords**: pantograph differential operators, solvable extension, spectrum, resolvent operator.

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