Approximation of the inverse elliptic problem with mixed boundary value conditions

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Abstract: In this study, we consider inverse problem of finding $u(t,x)$ and $p(x)$ for the multidimensional elliptic equation with the following Dirichlet-Neumann boundary value conditions

$$\begin{cases} -u_{tt} - \sum_{r=1}^{n} \left(a_r(x) u_{x_r}\right)_{x_r} + \delta u(t,x) = f(t,x) + p(x), \\ \qquad x = (x_1, \dots, x_n) \in \Omega,\ 0 < t < T, \\ u(0,x) = \varphi(x), u(T,x) = \psi(x), u(\lambda, x) = \xi(x), x \in \overline{\Omega} \\ \qquad \frac{\partial u(t,x)}{\partial \vec{n}} = 0, x \in S_1, u(t,x) = 0, x \in S_2. \end{cases} \qquad (1)$$

Here $a_r(x)$ $(\boldsymbol{x} \in \Omega)$, $\varphi(x)$, $\psi(x)$, $\xi(x)$ $(x \in \overline{\Omega})$, $f(\mathrm{t},\mathrm{x})$ $(t \in (0,T), x \in \Omega)$ are given smooth functions, $a_r(x) \geq 0, \Omega = (0,l) \times \dots \times (0,l)$ is the open cube in the n-dimentional Euclidian space with boundary $S = S_{\mathbf{1}} \cup S_2, \overline{\Omega} = \Omega \cup S$, $0 < \lambda < T$ and $\delta > 0$ are known constants.

Well-posedness of the inverse problem (1) follows from results of abstract theorems [2]. In the present work, the first and second orders of accuracy difference schemes for the approximate solution of inverse problem (1) are presented. Stability, almost stability and coercive stability estimates for the solution of these difference schemes are obtained. The algorithm for approximate solution is tested in a two-dimensional case.

Keywords: difference scheme, overdetermination, well-posedness, stability, coercive stability.

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