Duality theorems for the noncommutative spaces

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Abstract: In this work, we introduce the noncommutative $H_p(A,l_\infty)$ and $H_p(A,l_1)$ spaces, and give results on duality.

Let M be a finite von Neumann algebra equipped with a normal faithful tracial state τ . Let D be a von Neumann subalgebra of M, and let $\phi: M \to D$ be the unique normal faithful conditional expectation such that finite subdiagonal algebra of M with respect to ϕ is a w* -closed subalgebra A of M satisfying the following conditions:

(i) $A+A^*$ is w *-dense in M;
(ii) ϕ is multiplicative on A;
(iii) $A \cap A^* = D$,

where A^* is the family of all adjoint elements of the element of A. The algebra D is called the diagonal of A (see [1],[3]). We define $H_p(A,l_\infty)$ and $H_p(A,l_1)$ spaces by a similar way as in [3].

Theorem. Let $1 \le p < \infty$ such that $1/p + 1/p' = 1$. Then

(i) $H_p(A;l_1)^* = L_{p'}(M;l_\infty)/H_{p'}^0(A;l_\infty)^*$ and $(L_p(M;l_1)/H_p^0(A;l_1))^* = H_{p'}(A;l_\infty)$ isometrically via the following duality bracket

$$((x_n),(y_n)) = \sum_{n=1}^{\infty} \tau(y_n^* x_n)$$

for $x \in H_p(A;l_1)$ and $y \in H_{p'}(A;l_\infty)$.

Keywords: von Neumann algebra, subdiagonal algebras, vector valued noncommutative Hardy spaces, duality theorems.

References:

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