The third-order polynomial bundle and eigenfunctions

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Abstract: We consider the bundle [1] of the third order with involution $Au = (\alpha^2 - 1)u'''(x) - \alpha\lambda^2 u'(-x) + \lambda^3 u(-x), \quad \alpha \neq 0, \alpha \neq \pm 1$
in the space $L_2(-1,1)$ and the corresponding it equation

$$(\alpha^2 - 1)u'''(x) - \alpha\lambda^2 u'(-x) + \lambda^3 u(-x) = 0, -1 < x < 1. \qquad (1)$$

Together with equation (1) , we study the following spectral problem [2]

$$-u'(-x) + \alpha u'(x) = \lambda u(x), \ -1 < x < 1, \ \alpha \neq 0, \alpha \neq \pm 1 \qquad (2)$$

with some boundary conditions. The equation (2) contains involution.
The connection between the eigenfunctions of the bundle A and the spectral problem (2) is established in the following theorem.
Theorem 1. The eigenfunctions of the spectral problem (2) are the eigenfunctions of the bundle A and satisfies the equation (1).
The basis property of subsystem eigenfunctions of the operator bundle A with boundary conditions is established in the following theorem.

$$u(-1) = u(1), \ u'(-1) = u'(1), \ u''(-1) = u''(1). \qquad (3)$$

Theorem 2. The following functions

$$u_k(x) = -\sqrt{\frac{1-\alpha}{\alpha+1}} cosk\pi x + sink\pi x, k = 0, \pm 1, \pm 2, \ldots,$$

are eigenfunctions of the operator bundle (1), (3), corresponding to the
$\lambda_k = -\sqrt{1-\alpha^2}k\pi, \ k = 0, \pm 1, \pm 2, \ldots,$ and form a Riesz basis in $L_2(-1,1)$.

Keywords: operator bundle, eigenfunctions, basis, differential operator with involution**.**

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