

On a first order partial differential equation with the nonlocal boundary condition

Allaberen Ashyralyev^{a,b}, Sueda Nur Tekalan^c, Abdullah Said Erdogan^d

^{a,c,d}Department of Mathematics, Fatih University, Turkey

^bDepartment of Mathematics, ITTU, Turkmenistan

^aaashyr@fatih.edu.tr, ^csuedatekalan@gmail.com, ^daserdogan@fatih.edu.tr

Abstract: In this study, we consider the initial value problem

$$\begin{cases} u_t(t,x) + a(x)u_x(t,x) + \delta u(t,x) = f(t,x), 0 < x < l, 0 < t < T, \\ u(t,0) = u(t,l), 0 \leq t \leq T, \\ u(0,x) = u_0(x), 0 \leq x \leq l \end{cases} \quad (1)$$

for the first order partial differential equation with the nonlocal boundary condition. Here, $b \geq a(x) \geq a > 0$ and $u_0(x) (x \in (0,l))$ and $f(t,x) ((t,x) \in (0,T) \times (0,l))$ are given smooth functions and they satisfy every compatibility conditions which guarantees problem (1) has a smooth solution $u(t,x)$. The positivity of the space operator A generated by problem (1) in the space C with maximum norm is established. The structure interpolation spaces of the space operator A are investigated. The positivity of this space operator in Holder spaces is established.

The finite difference method for the approximate solution of problem (1) is presented. The positivity of the difference analogy of the space operator generated by this problem in the space C with maximum norm is established. The structure interpolation spaces generated by this difference operator are studied. The positivity of this difference operator in Holder spaces is established.

Following some mathematical models [1-3], in application the Mathematical Modelling of lake pollution is described and an example is given.

Keywords: stability analysis, difference schemes, pollution model.

References:

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