Well-posedness of difference scheme for elliptic-parabolic equations in Holder spaces without a weight

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Abstract: In the present paper, we are interested in studying the following second order of accuracy difference scheme for the solution of the elliptic-parabolic equation with nonlocal boundary condition

$$\begin{cases} -\frac{u_{k+1}-2u_k+u_{k-1}}{\tau^2}+Au_k=g_k, g_k=g(t_k), \\ t_k=k\tau, 1\le k\le N-1, N\tau=1, \\ \frac{u_k-u_{k-1}}{\tau}=-\frac{1}{2}(Au_k+Au_{k-1})=f_k, f_k=f\left(t_{k-\frac{1}{2}}\right), \\ t_{k-\frac{1}{2}}=\left(k-\frac{1}{2}\right)\tau, -(N-1)\le k\le 0, \\ \mathrm{u}_2-4\mathrm{u}_1+3\mathrm{u}_0=-3\mathrm{u}_0+4\mathrm{u}_{-1}-\mathrm{u}_{-2}, \mathrm{u}_N=\mathrm{u}_{-N}+\mu. \end{cases} \quad (1)$$

Theorem on well-posedness of this problem in Holder spaces without a weight is given. In an application, coercivity estimates in Holder norms for approximate solution of a nonlocal boundary value problem for elliptic-parabolic differential equation are obtained.

Keywords: difference scheme, elliptic-parabolic equation, coercivity inequalities, well-posedness.

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