On the solvability of boundary value problems for the nonhomogeneous polyharmonic equation in a ball

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Abstract: Let m be a positive integer. We consider in n-dimensional unit ball $\Omega = \{x : |x| < 1\} \subseteq R^n$ the nonhomogeneous polyharmonic equation (PHE)

$$\Delta_x^m u(x) = f(x), \qquad (1)$$

with the boundary conditions

$$\left. \frac{\partial^{k_j}}{\partial n_x^{k_j}} u \right|_{x \in \partial\Omega} = \varphi_j(x), \ x \in \partial\Omega, \ j = \overline{1,m}; \ 0 \le k_1 < k_2 < \ldots < k_m \le 2m-1. \qquad (2)$$

Usually, the existence of regular solutions to the original data $f(x)$, $\varphi_1(x), \varphi_2(x), \ldots, \varphi_m(x)$ imposes limitations of two types [1,2]: (i) some loss of the smoothness; (ii) certain conditions such as orthogonality to the solutions of the homogeneous adjoint equation.

In this paper, we describe criterion for the solvability of the problem (1) - (2) in the initial terms [3] in particular, Neumann problem (when $k_1 = 1, k_2 = 2, \ldots k_m = m$). We give representation of the Green function of the Dirichlet problem (when $k_1 = 0, k_2 = 1, \ldots k_m = m-1$) for the PHE in a ball without restrictions on the number of spatial variables and the order of the equation [4], as well as built some classes correct boundary value problems in bounded domains.

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Keywords: polyharmonic equation, regular solution, Dirichlet problems, solvability of boundary value problems.

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