On third order of accuracy stable difference scheme for multipoint NBVP of the hyperbolic type

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Abstract: We consider the following multipoint nonlocal boundary value problem (NBVP)

$$\begin{cases} \frac{\partial^2 u(t,x)}{\partial t^2} - \sum_{r=1}^{m} (a_r(x) u_{x_r})_{x_r} = f(t,x), \\ x = (x_1, \ldots, x_m) \in \Omega_h, \; 0 < t < 1, \\ u^h(0,x) = \sum_{j=1}^{n} \alpha_j u^h\left(\lambda_j, x\right) + \phi^h(x), x \in \overline{\Omega}_h, \\ u_t^h(0,x) = \sum_{j=1}^{n} \beta_j u_t^h(\lambda_j, x) + \psi^h(x), x \in \overline{\Omega}_h, \\ u^h(t,x) = 0, \; x \in S. \end{cases} \tag{1}$$

Here, $a_r(x)$, $(x \in \Omega)$, $\phi(x), \psi(x)$ $(x \in \overline{\Omega}_h)$ and $f(t,x)$ $(t \in (0,1), \; x \in \Omega_h)$ are given smooth functions and $a_r(x) \geq a > 0$, $\Omega = (0,l) \times \ldots \times (0,l)$ is the open cube in the m-dimensional Euclidean space with the boundary $S = S_1 \cup S_2$, $\bar{\Omega} = \Omega \cup S$, $0 < \lambda < T$ and $\delta > 0$ are known constants.
In this work, the third order of accuracy difference scheme for approximately solving problem (1) generated by the integer power of the operator is presented. Stability estimates for the solution of these difference schemes are obtained. The theoretical statements are supported by the numerical results using MATLAB.

Keywords: multipoint nonlocal boundary value problem, difference schemes, convergence, stability, numerical analysis.

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