On fourth order of accuracy stable difference scheme for multipoint NBVP of the hyperbolic type

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Abstract: In this work, we consider the multipoint NBVP

$$\begin{cases} \frac{\partial^2 u(t,x)}{\partial t^2} - \sum_{r=1}^{m} (a_r(x) u_{x_r})_{x_r} = f(t,x), \\ x = (x_1, \ldots, x_m) \in \Omega_h, \ 0 < t < 1, \\ u^h(0,x) = \sum_{j=1}^{n} \alpha_j u^h(\lambda_j, x) + \phi^h(x), x \in \overline{\Omega}_h, \\ u_t^h(0,x) = \sum_{j=1}^{n} \beta_j u_t^h(\lambda_j, x) + \psi^h(x), x \in \overline{\Omega}_h, \\ u^h(t,x) = 0, \ x \in S. \end{cases} \quad (1)$$

Here, $a_r(x), (x \in \Omega), \phi(x), \psi(x) \ (x \in \overline{\Omega}_h)$ and $f(t,x) \ (t \in (0,1), \ x \in \Omega_h)$ are given smooth functions and $a_r(x) \geq a > 0$, $\Omega = (0,l) \times \ldots \times (0,l)$ is the open cube in the m-dimensional Euclidian space with the boundary $S = S_1 \cup S_2$, $\overline{\Omega} = \Omega \cup S$, $0 < \lambda < T$ and $\delta > 0$ are known constants.

The fourth order of accuracy difference scheme for approximately solving problem (1) generated by the integer power of the operator is presented. Stability estimates for the solution of these difference schemes are obtained. The theoretical statements are supported by the numerical results using MATLAB.

Keywords: multipoint nonlocal boundary value problem, difference schemes, convergence, stability, numerical analysis.

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