Stability and convergence results for modified Jungck-Picard-S iterative method when applied to asymptotically nonexpansive mappings

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Abstract: $\mathbb{N}$ will denote the set of natural numbers including zero over the course of this study. Let C be a nonempty subset of a Banach space B and $S, T: C \to C$ be two mappings with $T(C) \subseteq S(C)$. Then, we denote the set of all fixed points of T by F_T and the set of common fixed points of S and T by F.

It is now considered indisputable that iteration methods are among the most important and useful mathematical tools which are used in solving a wide variety of problems arise in mathematics and other branches of science.

In this study, we introduce a Modified Jungck-Picard-S hybrid type iterative methods as follows:

$$\begin{cases} x_0 \in C, \\ Sx_{n+1} = T^n y_n, \\ Sy_n = (1 - \alpha_n^0)T^n x_n + \alpha_n^0 T^n z_n, \\ Sz_n = (1 - \alpha_n^1)Sx_n + \alpha_n^1 T^n x_n, n \in \mathbb{N}, \end{cases} \tag{1}$$

where $\left\{\alpha_n^i\right\}_{n=0}^{\infty}, i = \overline{0,1}$, are real sequences in $[0,1]$ satisfying certain control condition(s).

We establish weak stability, weak convergence and strong convergence results for a pair of Jungck asymptotically nonexpansive mappings with the help of the Modified Jungck-Picard-S iterative procedure. An illustrative example is also discussed to show that the iterative procedure (1) converges faster than some iterative procedures in the existing literature.

Keywords: modified Jungck-Picard-S iteration method, Jungck asymptotically nonexpansive mappings, fixed points, weak and strong convergence, weak stability.

References:
[1] M. Aggarwal, R. Chugh, S. Kumar, Stability and Convergence of Jungck Modified S-iterative Procedures, International Journal of Engineering and Innovative Technology (IJEIT), vol. 3, no. 2, pp. 223-231, 2013.
[2] F. Gursoy, V. Karakaya, A Picard-S hybrid type iteration method for solving a differential equation with retarded argument, arXiv preprint arXiv:1403.2546, 2014.

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