The Cauchy problem for a doubly nonlinear parabolic equation with inhomogeneous density and sources

Mirsaid Aripov [a], *Zafar Rakhmonov* [b]
[a,b] *National University of Uzbekistan, Uzbekistan*
[a] *mirsaidaripov@mail.ru*, [b] *zraxmonov@inbox.ru*

Abstract: In $Q_T = R_+^N \times (0,T), \quad T>0, \quad N \geq 1$, we consider the Cauchy problem for doubly nonlinear parabolic equation with a source and an inhomogeneous density:

$$|x|^m \frac{\partial u}{\partial t} = div\left(|x|^k \left|\nabla u^l\right|^{p-2} \nabla u^l\right) + |x|^m u^q, \tag{1}$$

$$u(x,0) = u_0(x) \geq 0, \; x \in R_+^N, \tag{2}$$

where m, k, $l>0$, $p>2$ are given numerical parameters.

Equation (1) arises in the mathematical modeling of diffusion processes in nonlinear media. The problems of fluid flow through porous layers, in the dynamics of biological populations, and in some other areas are studied in [1, 2]. Various properties of the solutions of the Cauchy problem and boundary value problems for (1) are intensively investigated by many authors [2-7].

On the basis of the self-analysis, the global solvability and nosolvability (Lions' problem) of the Cauchy problem (1), (2) is proved, including in the case of $l = p$, the asymptotic behavior of solutions with compact support and vanishing at infinity of solutions are obtained. The initial approximations for numerical calculations is proposed. It leads to the exact solution with rapid convergence.

Keywords: global solvability, nosolvability, asymptotic, numerical analysis.

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